<u>Gukov</u> hep-th/0512298

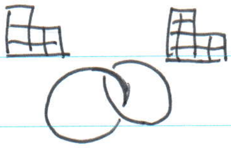
arXiv/0705.1368

triply-graded invariants

$$\mathcal{H}^{i,j,k}_{X,R_1,\dots,R_\ell}(L)$$

$\uparrow$ $X = A, B, C, D$

$\ell = \#$ of components of L



easily computable

triply graded theory

$\mathcal{H}^{\text{Kauffmann}}_{i,j,k}$ s.t. $\chi(\mathcal{H}^{\text{Kauff}}_{i,j,k}) = \text{Kauffman}(a,q)$

$\dim(\mathcal{H}^{\text{Kauff.}}_{i,j,k}) < \infty$

* differentials d_N $N>0$

$$\deg(d_N) = (\underset{a}{-1}, \underset{q}{N-1}, \underset{t}{-1})$$

$$(\mathcal{H}^{\text{Kauff}}_{*}, d_N) \cong HSO_N(K)$$

differentials $\longleftrightarrow$ deformations of $W(x_i)$

$W = x^{N'-1} + xy^2$ $\delta W = x^N$ $N' > N$

$\cdots SO_N$

* differentials d_N $N<0$

$$\deg(d_N) = (-1, N-1, N-1)$$

$$(\mathcal{H}^{\text{Kauff}}_{*}, d_N) \cong HSp_N(\bar{K})$$

mirror image

京都大学大学院理学研究科数学教室

* "cancelling" differentials d_0, d_1, d_2

$\deg d_0 = (-1,-1,-2)$

$d_1 = (-2,0,-3)$

$d_2 = (-1,1,-1)$

$(\mathcal{H}^{\mathrm{Kauff}}, d_i)$ $i=0,1,2$

$\cong$ "trivial"

* "universal" differentials $d_{\rightleftarrows}$

correspond $\delta W = y^2$

$SO(N) \to SL(N-2)$

$\left\{ \begin{array}{ll} \deg d_{\leftarrow} & = (0,2,-1) \\ d_{\rightarrow} & = (0,2,1) \end{array} \right.$

a-grading is 0

$(\mathcal{H}^{\mathrm{Kauffmann}}_*, d_{\rightleftarrows})$

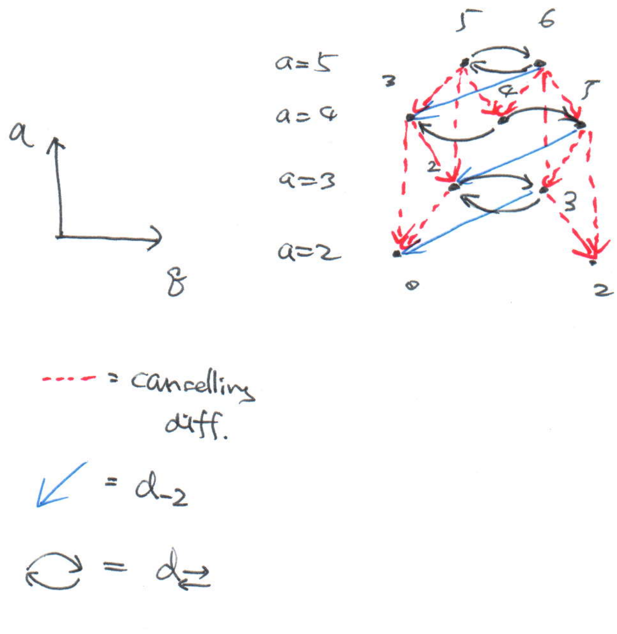
$\cong \mathcal{H}^{\mathrm{HOMFLY}}_*$

<u>Ex.</u>

$\dim \mathcal{H}^{\mathrm{Kauff}}(3_1)$

$\parallel$

9



Gauge theory and categorification

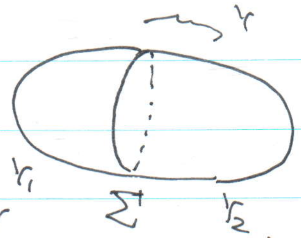
3D TQFT is a functor :

3-mfd $Y \longmapsto$ number $Z(Y)$

surface $\Sigma \longmapsto$ vector space $\mathcal{H}_\Sigma$

Heegard decompostion

$Y = Y_1 \cup_\Sigma Y_2$ $\qquad\qquad Z(Y) = \langle Y_1 | Y_2 \rangle$ in $\mathcal{H}_\Sigma$



$\langle Y_1 | \in \mathcal{H}_\Sigma \qquad \mathcal{H}_\Sigma \qquad | Y_2 \rangle \in \mathcal{H}_\Sigma$

4D gauge theory

gauge theory on $X \rightsquigarrow$ number $Z(X) \qquad = \chi(\underset{\text{moduli sp.}}{\mathcal{M}(X)})$

gauge theory on $\underset{\text{time}}{\mathbb{R}} \times Y \rightsquigarrow$ vector space $\mathcal{H}_Y \qquad = H^*(\mathcal{M}(Y))$

translation inv. under $\mathbb{R}$

gauge theory on $\mathbb{R}^2 \times \Sigma \rightsquigarrow$ category $\mathcal{F}(\Sigma) \qquad = \mathcal{F}(\Sigma)$

Use $\mathcal{M}(\Sigma)$

trans. inv. under $\mathbb{R}^2$

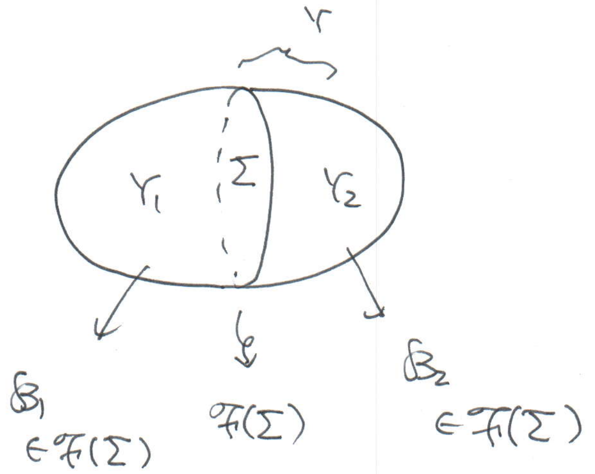
4	3	2
instanton	monople	vortex equ.

To get $\mathcal{F}(\Sigma)$

use topological twist : A $\cdots$ $\mathrm{Fuk}(\mathcal{M}(\Sigma))$

B $\cdots$ $D^b(\mathcal{M}(\Sigma))$

boundary condition



$\mathcal{B}_1, \mathcal{B}_2$ bdry states

$\rightsquigarrow \mathcal{H}_Y$: open strings between them

$$\mathcal{H}_Y = \begin{cases} HF_*^{symp}(\mathcal{M}, \mathcal{L}_1, \mathcal{L}_2) \\ \mathrm{Ext}^\bullet(\mathcal{B}_1, \mathcal{B}_2) \end{cases}$$

<u>Ex.</u> Donaldson-Witten theory

$\mathcal{H}_Y$ = instanton Floer homology $HF_*^{inst}(Y)$

$\mathcal{M}(\Sigma)$ = moduli space of flat G-bundle $\mathcal{M}^G_{flat}(\Sigma)$

$$HF_*^{inst}(Y) \cong HF_*^{symp}(\mathcal{M}^G_{flat}(\Sigma), \mathcal{L}_1, \mathcal{L}_2)$$

(Atiyah-Floer conjecture)

B-model より.

<u>surface operators</u>

operators in 4D gauge theory

supported on 2-dim'l surface $D \subset X$

- closed surface $D \subset X$ $\rightsquigarrow$ $Z(D, X)$ cf. Kronheimer–Mrowka Embedded surface ...
- $X = \mathbb{R} \times Y$, $D = \mathbb{R} \times K$ (Knot) $\rightsquigarrow$ $\mathcal{H}_{Y,K}$

$$\begin{array}{ccc} X & = & \mathbb{R} \times Y \\ \| & & \cup \\ D & = & \mathbb{R} \times K \end{array}$$

- $X = \mathbb{R}^2 \times \Sigma$, $D = \mathbb{R}^2 \times pt$ $\rightsquigarrow$ $\mathcal{F}(\Sigma, p, \text{parameters})$

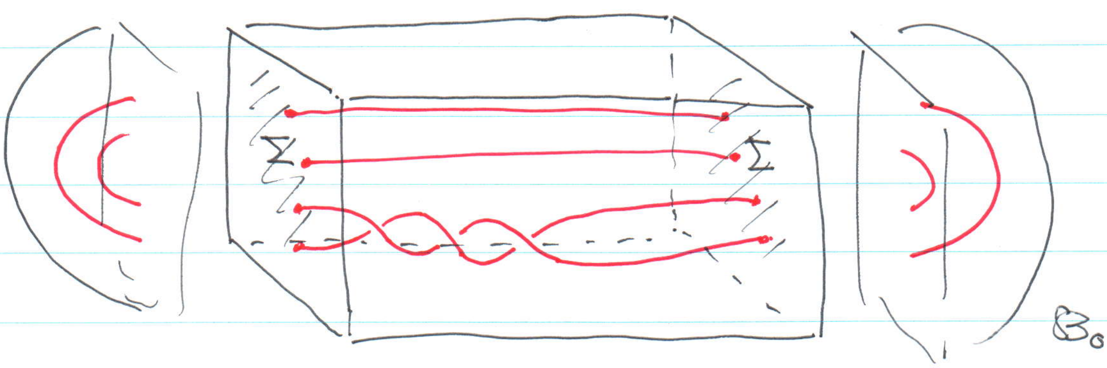
Physics
(Wilson line
(t'Hooft line

<u>Note</u> : the mapping classgroup of Σ acts on $\mathcal{F}(\Sigma)$

e.g. if $\Sigma = \mathbb{C} \setminus \{p_1, \dots, p_n\}$

$$\Longrightarrow \underbrace{Br_n}_{\text{braid group}} = \pi_1(\mathrm{Conf}^n(\mathbb{C})) \curvearrowright \mathcal{F}(\Sigma)$$

$\beta \mapsto \phi_\beta$



$$\phi_\beta : \mathcal{F}(\Sigma) \longrightarrow \mathcal{F}(\Sigma)$$

$$\mathcal{H}_K = \begin{cases} HF_*^{symp}(\mathcal{M}, \mathcal{B}_0, \phi_\beta(\mathcal{B}_0)) & \text{A-model} \\ \mathrm{Ext}^*(\mathcal{B}_0, \phi_\beta(\mathcal{B}_0)) \end{cases}$$

N=4 YM .... affine Hecke alg categorification (ramified Higgs bdle moduli sp.